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Stochastic Modeling

Exercises for 29/01/2002

-1-

(1) (a) Show the Formula of Bayes: For $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$ it holds

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A)\mathbb{P}(B|A)}{\mathbb{P}(B)}.$$

(b) Deduce the Theorem of Bayes:

$$\mathbb{P}(A_i|B) = \frac{\mathbb{P}(B|A_i)\mathbb{P}(A_i)}{\sum_{j=1}^n \mathbb{P}(B|A_j)\mathbb{P}(A_j)},$$

where $\mathbb{P}(A_i) > 0, i = 1, \dots n$; $\mathbb{P}(B) > 0$, and $\Omega = \bigcup_{k=1}^n A_k$ with $A_k \cap A_l = \emptyset$ if $k \neq l$.

(c) There are two coins, one fair, one unfair (with respect to tossing):

$$\mathbb{P}(\text{ fair coin} = \text{head}) = \frac{1}{2},$$

$$\mathbb{P}(\text{ unfair coin} = \text{head}) = \frac{1}{3}.$$

One chooses one of these coins, tosses it, and it shows 'head'. Show using the Theorem of Bayes that it holds

$$\mathbb{P}(\text{ fair coin was chosen}|\text{head}) = \frac{3}{5}$$

assuming

$$\mathbb{P}(\text{ fair coin was chosen}) = \mathbb{P}(\text{ unfair coin was chosen}) = \frac{1}{2}.$$

(2) Compute the expectation of a random variable

(a) $f : \Omega \to [a, b]$ which has a uniform distribution (restricted on the interval $[a, b]$):

$$l_{[a,b]} := \frac{1}{b-a} l(\cdot \cap [a, b])$$

(b) $g : \Omega \to \{0, 1, \dots\}$ which has a Poisson–distribution:

$$\Pi_\lambda := \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} \delta_k$$

(c) $h : \Omega \to \{0, 1, \ldots, n\}$ which has a binomial distribution:

$$B_{n,p} := \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{(n-k)} \delta_k$$

(3) If one plays at dice one assumes that the probability to get $1, 2, \ldots$ or 6 is always $\frac{1}{6}$. Prove that the probability that a dice shows k times 6 if it is rolled n times ($0 \le k \le n$) is a binomial distribution. (Which special $\Omega, \mathcal{F}$, and $\mathbb{P}$ fit this situation?)

(4) The random variables f and g are independent and Poisson–distributed with parameters λ_1 and λ_2, respectively. Show that $f + g$ has again a Poisson distribution by computing

$$\mathbb{P}(f + g = k) = \sum_{l=0}^{k} \mathbb{P}(f = l, g = k - l) =$$

What is the parameter of this Poisson distribution ?