Bridging socioeconomic pathways of carbon emission and credit risk

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Introduction

- The climate change such as global warming and extreme events has become an imminent worldwide challenge.
- Climate risks in finance
  - Physical risks : economic cost and financial losses resulting from the increasing severity and frequency of extreme climate change-related weather events
  - Transition risks : risks related to the process of adjustment towards a low-carbon economy, it concerns social and political instability of policiesas well as technological changes
- Banks are encouraged to transition their portfolios to low-carbon assets
  - Network for Greening Financial System (NGFS)
  - Which quantitative methodology for an ecological portfolio transition? Which variables?
  - Which optimization criterion for better transition paths?
- This talk: modelling of the transmission channel "transition risk" to "credit risk", and aggregate in a large portfolio (on-going work)

RCP Projections of greenhouse gas emissions

- Intergovernmental Panel on Climate Change (IPCC) published RCPs (Representative Concentration Pathways) by summarizing different potential scenario of global warming.
- The increase of global mean surface temperature by 2100 : 0.3°C-1.7°C under RCP2.6; 1.1°C-2.6°C under RCP4.5; 1.4°C-3.1°C under RCP6.0; and 2.6°C-4.8°C under RCP8.5.

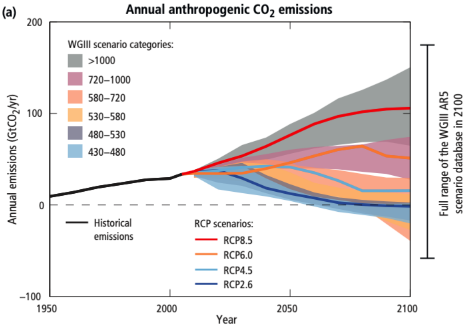

Figure: Source: Fifth Assessment Report (AR5) of IPCC

EU climate action and target

- The Paris Agreement has set the idealized objective for a global warming around only 1.5^{o}C before 2100.
- The European Union commission has planed to cut emissions by at least 55% by 2030 and aims to become the world's first climate-neutral continent by 2050.
- A series of legislation and policies have been adopted to achieve the climate-neutrality objective, including European Climate Law and Pact, and the EU Emissions Trading System (ETS).
- In this context, many other possible scenarios, such as the Shared Socioeconomic Pathways (SSPs) for the CMIP6 project, are developed in scientific literature according to more detailed socio-economic and ecological criteria, and also for different sectors, countries etc.

Shared Socioeconomic Pathways (SSPs)

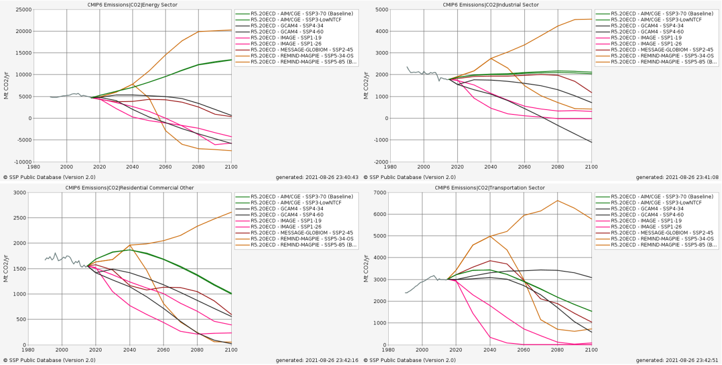

Figure: Historical and scenario-based CO2 emission, from 1980 to 2100, in Mt/yr in the OECD, according to the activity sectors: Energy (top left), Industry (top right), Residential Commercial (bottom left), Transportation (bottom right).

Our work

- We consider firms who are facing climate transition risks (no physical risk here) towards a low-carbon production pattern.
- Main objective: model and quantify how different SSPs projection scenarios impact the credit risk.
- Given an emission scenario (by SSP), a firm aims to determine its effective emission level under the double criteria of maximizing the profit and respecting the emission target.
- Firm's climate-related value process is deduced and the default is modelled by the Merton model
- Compute the (semi-explicit) default probability and analyse the impact of input SSPs scenarios.
- Allow for large portfolio extension for cumulative losses.

Model Setup

- Let the probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with a filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ represent the market.
- Consider a firm whose production is given by the SDE

$$dP_t = P_t \left(\mu\left(t, P_t, \gamma_t\right) dt + \sigma dW_t\right), \quad P_0 > 0,$$

where $\sigma > 0$ and

  - γ_t is the instantaneous emission rate
  - $\mu : (t, x, y) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ satisfies the local Lipschitz condition on x and is of classe C^1 on (x, y)
  - $\partial\mu_x < 0$: overproduction will reduce the production rate
  - $\partial\mu_y > 0$: empirical studies show that the effect of emissions on production growth is positive.

Emission benchmark

- Let $t \to e_t$ be an emission trajectory for the firm (e.g. a SSP projection). It will serve as a benchmark of the effective emission $t \to \gamma_t$.
- Exceeding the benchmark can induce penalty or losses to the firm (carbon taxes or purchase of extra allowance through ETS)
- The penalty constraints are specified by using loss functions related to risk measures
- Let $\ell : \mathbb{R} \to \mathbb{R}$ be a loss function which is increasing and convex with initial value $\ell(0) = 0$ and quadratic growth, i.e., $\ell(x) = \mathcal{O}(|x|^2)$ as $|x| \to +\infty$.

Production profit vs emission constraint

- Firm's goal: maximize its production profit and manage the effective emission with trajectory constraints

$$J_\infty(\gamma) := \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\pi(P_t) - \mathcal{C}(\gamma_t) - \ell(\gamma_t - e_t)\right) dt\right] \quad (1)$$

where

- $r \geq 0$ a constant discount rate
- $\pi : \mathbb{R}_+ \to \mathbb{R}$: profit function increasing and concave, of class C^1, and satisfying the Inada conditions $\lim_{x \to 0+} \pi'(x) = +\infty$ and $\lim_{x \to +\infty} \pi'(x) = 0$
- $\mathcal{C} : \mathbb{R}_+ \to \mathbb{R}_+$: emission-related cost function increasing and convex meaning that higher emissions induce over-usage deterioration.

Optimization problem

- Aim to solve
$$\widehat{J} = \sup_{\gamma \in \mathcal{A}} J_\infty(\gamma)$$
where $\mathcal{A}$ is the admissible strategy set such that $\mathbb{E}[(\int_0^\infty e^{-\eta t}\gamma_t dt)^2] < +\infty$ for some $\eta \in (0, r)$.
- Find the optimal effective emission strategy $\widehat{\gamma}$ of the firm.

Resolution by Pontryagin's maximum principle

- The optimisation problem can be solved by adopting the Pontryagin's maximum principle for the optimal strategy by using the method of Lagrange multipliers applied to a constrained optimization problem.
- Introduce the following change of variables: the log-production $p_t := \log P_t$ which solves

$$dp_t = \overline{\mu}(t, p_t, \gamma_t)dt + \sigma dW_t,$$

  with $\overline{\mu}(t, x, y) := \mu(t, e^x, y) - \frac{1}{2}\sigma^2$ and the auxiliary cost function

$$\overline{\pi}(x) := \pi(e^x)$$

Optimal effective emission

- We characterize the solution of the infinite problem $J_\infty(\widehat{\gamma})$.
- Let

$$Y_t = \mathbb{E}\left[\int_t^\infty e^{-ru+\int_t^u \partial_x \overline{\mu}(t,p_s,\gamma_s)ds} \overline{\pi}'(p_u)du \middle| \mathcal{F}_t \right]$$

- The optimal effective emission $\widehat{\gamma}$ is then given as the solution of the following equation

$$\mathcal{C}'(\widehat{\gamma}_t) + \ell'(\widehat{\gamma}_t - e_t) = e^{rt}\partial_y \overline{\mu}(t, \widehat{p}_t, \widehat{\gamma}_t)\widehat{Y}_t$$

- Note that $\lim_{t\to+\infty} Y_t = 0$.

Alternative formulation with finite horizon

- Consider a final horizon time $T > 0$ (e.g. 2050) and include an extra cumulative emission penalty.
- Define cumulative benchmarked and effective emission

$$E_t = \int_0^t e_s ds, \qquad \Gamma_t = \int_0^t \gamma_s ds.$$

- The objective function is

$$J_T(\gamma) := \mathbb{E}\Big[\int_0^T e^{-rt}\left(\pi(P_t) - \mathcal{C}(\gamma_t) - \ell_1(\gamma_t - e_t)\right) dt - e^{-rT}\ell_2\left(\Gamma_T - E_T\right)\Big]$$

- Solve

$$\widehat{J}_T = \sup_{\gamma \in \mathcal{A}} J_T(\gamma)$$

where $\mathcal{A}$ is the admissible strategy set such that $\mathbb{E}[\Gamma_T^2] < +\infty$

Optimal emission with finite time horizon

- The finite horizon problem can be solved in a similar way. The difference lies in the extra terminal constraint.
- The solution for $\widehat{J_T} = J_T(\widehat{\gamma})$ is characterized by the following linear BSDE

$$\begin{cases} dY_t^1 & = -\left(e^{-rt}\overline{\pi}'(p_t) + \partial_x \overline{\mu}(t, p_t, \gamma_t)\, Y_t^1\right) dt + dM_t^1, \\ Y_T^1 & = 0 \end{cases}$$

where M^1 is an $\mathbb{F}$-martingale, so that

$$\widehat{Y}_t^1 = \mathbb{E}\left[\int_t^T e^{-ru + \int_t^u \partial_x \overline{\mu}(t, \widehat{p}_s, \widehat{\gamma}_s) ds}\, \overline{\pi}'(\widehat{p}_u) du \middle| \mathcal{F}_t \right]$$

- The optimal emission $\widehat{\gamma}$ satisfies

$$e^{-rt}\left[\mathcal{C}'(\widehat{\gamma}_t) + \ell_1'(\widehat{\gamma}_t - e_t)\right] + \mathbb{E}\left[e^{-rT} \ell_2'\left(\widehat{\Gamma}_T - E_T\right) \middle| \mathcal{F}_t \right] = \partial_y \overline{\mu}(t, \widehat{p}_t, \widehat{\gamma}_t)\, \widehat{Y}_t^1$$

Emission-related credit risk

- Credit risk means the possibility and potential losses due to the incapacity of the firm to reimburse its debt obligations.
- In the structural approach of credit modelling, a firm defaults if its value is not sufficient to repay the debt liability.
- Our aim: analyse the emission impact on default probability.
- Describe the firm's value at a given date t by $V_t^{\widehat{\gamma}}$ and denote by L_t the liability value which will serve as the default barrier.
- The default probability in Merton model is defined as

$$DP_t = \mathbb{P}(V_t^{\widehat{\gamma}} < L_t)$$

closed-form formula can be obtained for certain model specifications.

Firm's value process

- Define the value process of the firm by the so-called "discounted cash flow" approach

$$V_t^\gamma = \mathbb{E}\left[\int_t^\infty e^{-r(u-t)}\left(\pi(P_u) - \mathcal{C}(\gamma_u) - \ell(\gamma_u - e_u)\right) du | \mathcal{F}_t\right].$$

  which is the conditional discounted value of all future cash flows depending on the effective emission γ.
- The firm will produce according to the optimal emission $\widehat{\gamma}$ and production $\widehat{P}$ from the optimization procedure.
- The value process $V_t^{\widehat{\gamma}}$ associated to the emission $\widehat{\gamma}$ achieves the firm's optimal value $\widehat{V}_t = \operatorname*{ess\,sup}_{\gamma \in \mathcal{A}(t,\nu)} V_t^\gamma$

Application with an explicit model

- To compute explicitly the value process, we choose respectively the profit, cost and penalty functions as

$$\pi(x) = Nx, \qquad \mathcal{C}(x) = \frac{x^2}{2} \qquad \text{and} \qquad \ell(x) = \omega \frac{(x_+)^2}{2},$$

  where
  - $N \geq 0$ the average price for one unit of production
  - $\omega \geq 0$ and the quadratic penalty to accentuate higher quantities of over-emission.
- Let the drift coefficient of the log-production p_t be

$$\overline{\mu}(t, x, y) = a + bx + cy,$$

  where
  - $a \geq 0$ corresponds to an average production level
  - $b \leq 0$ mean-reverting parameter that over-production may decrease the production ability
  - $c \geq 0$ describes the dependence of production w.r.t. emission

Optimal emission

- From the constrained infinite horizon optimization and supposing $r - b > 0$, we have

$$\widehat{\gamma}_t = \left(\mathcal{C}'(\cdot) + \ell'(\cdot - e_t)\right)^{-1}\left(c\int_t^{\infty} e^{(b-r)(u-t)}du\right)$$
$$= \min\left\{\frac{c}{r-b},\ \frac{1}{1+\omega}\left(\omega e_t + \frac{c}{r-b}\right)\right\}$$

- The critical value

$$\overline{\gamma} := \frac{c}{r-b}$$

  is attained when $\omega = 0$, in case without penalty.
- If $e_t \geq \overline{\gamma}$, optimal emission remains at the constant level $\overline{\gamma}$ (no effort for the company).
- If $e_t < \overline{\gamma}$, meaning a stricter mitigation plan, the optimal emission is an affine function of the benchmark.

Default probability

- Given a SSP emission scenario and the associated optimal emission $\widehat{\gamma}$, the firm's value is

$$V_t^{\widehat{\gamma}} = N \int_t^\infty e^{-r(u-t)} \mathbb{E}\left[\widehat{P}_u | \mathcal{F}_t\right] du - \int_t^\infty e^{-r(u-t)} \left(\mathcal{C}(\widehat{\gamma}_u) + \ell(\widehat{\gamma}_u - e_u)\right) du$$
$$=: h(t, \widehat{p}_t)$$

  where $h(\cdot, \cdot)$ is a deterministic function.
- The default probability rewrites as

$$\mathbb{P}(V_t^{\widehat{\gamma}} \leq L_t) = \mathbb{P}(\widehat{p}_t \leq (h(t, \cdot))^{-1}(L_t))$$
$$= \Phi\left(\frac{(h(t, \cdot))^{-1}(L) - e^{bt} p_0 - m_{t,0}}{\sigma_{t,0}}\right),$$

  since the optimal log-production $\widehat{p}_t$ is a Gaussian process $\widehat{p}_t \sim \mathcal{N}(e^{bt} p_0 + m_{t,0}, \sigma_{t,0}^2)$ and Φ is the c.d.f. of $\mathcal{N}(0, 1)$.

Numerical illustration

- We illustrate relevant results for the Energy sector.
- The input are SSPs annual historical and future projection of CO2 emissions from 2015 to 2100
- We consider 5 different emission benchmark scenarios (including 3 baseline scenarios and 2 new pathways) and deduce corresponding default probability.
- The liability boundary L_t is specified at the level where there is no climate impact by

$$\mathbb{P}(\widehat{V}_t^{\mathrm{ref}} \leq L_t) = 1 - e^{-\lambda_{\mathrm{ref}} t},$$

where λ_{ref} is a reference value for default intensity chosen to be 3%, and $\widehat{V}_t^{\mathrm{ref}}$ corresponds to the optimal value without emission constraint, i.e., $\omega = 0$.

Energy sector

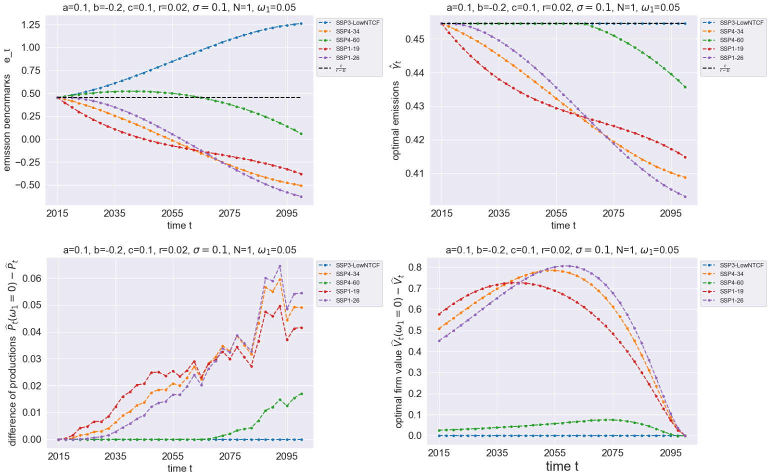

Figure: SSPs emission scenarios e_t up to 2100 (top left), Optimal effective emission $\widehat{\gamma}_t$ (top right), Production difference $\overline{P}_t(\omega=0)-\widehat{P}_t$ (bottom left), Value process difference $\widehat{V}_t(\omega=0)-\widehat{V}_t$ (bottom right).

Default probability and intensity for Energy sector

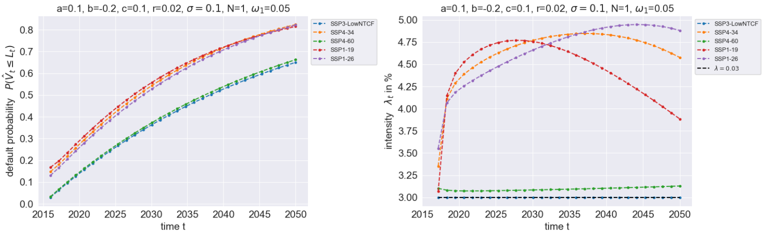

Figure: Default probability up to 2050 (left), Default intensity (right).

- The emission reduction projection has an instantaneous impact on default probability and intensity of the firm:
    - a larger mitigation scenario may imply an increase in the default intensity
    - facing a stricter constraint, the firm chooses to reduce its production and the firm's value decreases accordingly
    - without emission effort, the default intensity remains at the initial level

The aggregation of loss distribution

The cumulative loss of a portfolio is given by

$$\mathcal{L} = \sum_{i=1}^{n} \ell_i \, Y_i$$

- n: number of obligors ($n \geq 10^5$ or 10^6)
- ℓ_i: loss given default of the i^{th} obligor
- $Y_i = 1$ if the i^{th} obligor defaults or 0 otherwise
- In previous model, $Y = 1_{\{\hat{p}_t \leq (h(t,\cdot))^{-1}(L(t))\}}$ with optimal log-production $\hat{p}_t$ a Gaussian process

Our objective:

- Extend the previous model for multiple default times which depend on SSP scenarios
- Compute the loss distribution for credit risk measures

Difficulty: any Monte-Carlo (MC) simulation scheme is hugely time-consuming.

Gaussian factor model

- For large credit portfolio, defaults are generically modeled by $Y_i = 1_{\{X_i \leq c_i\}}$
  - c_i the default threshold for the ith obligor
  - Dependence between obligors achieved through the correlated stochastic factors X_i
- More precisely, for one-factor Gaussian copula,

$$X_i = \rho_i Z + \sqrt{1 - \rho_i^2}\epsilon_i$$

  - $Z \sim \mathcal{N}(0, 1)$ systemic risk factor (economy).
  - $(\epsilon_i)_{i=1\cdots n} \sim \mathcal{N}(0, 1)$ idiosyncratic risks i.i.d and independent from Z.
  - ρ_i : correlation parameter.
- For multi-factors: as many Z as different sectors
- To simplify now: single factor and positive correlation.

Extension of SSP-related credit model

- We generalize the previous model to n firms whose production is given by

$$dP_t^i = P_t^i\Big(\mu_i(t, P_t^i, \gamma_t^i)dt + \sigma_i dW_t^i\Big), \quad P_0^i > 0$$

  and decompose $dW_t^i = \rho_i dB_t + \sqrt{1-\rho_i^2}\, dB_t^i$
- Using previous optimization results, the total loss rewrites as

$$\begin{aligned}
\mathcal{L}_t &= \sum_{i=1}^n \ell_i \, 1_{\{\hat{p}_t^i \le (h^i(t,\cdot))^{-1}(L_i(t))\}} \\
&= \sum_{i=1}^n \ell_i \, 1_{\left\{m_t^i + \int_0^t a_s^i dB_s^i \le \int_0^t b_s^i dB_s\right\}}
\end{aligned}$$

  with m^i, a^i and b^i some explicit and adapted processes

The Wiener (polynomial) chaos expansion

- Let $Z \sim \mathcal{N}(0,1)$ and a measurable function $\varphi : \mathbb{R} \to \mathbb{R}$, s.t. $\mathbb{E}\left[\varphi^2(Z)\right] < +\infty$. Then $\varphi(Z)$ can be decomposed into L_2 as

$$\varphi(Z) = \sum_{k=0}^{\infty} \alpha_k \mathrm{He}_k(Z), \quad \alpha_k = \mathbb{E}\left[\varphi(Z)\mathrm{He}_k(Z)\right]/k!.$$

  where He_k : Probabilistic Hermite polynomial of degree $k \in \mathbb{N}$.
- Three-term recurrence relation :

$$\mathrm{He}_0(x) = 1,\ \mathrm{He}_1(x) = x,\ \mathrm{He}_{k+2}(x) = x\mathrm{He}_{k+1}(x) - (k+1)\mathrm{He}_k(x).$$

- Theorem

  The indicator function has an explicit Wiener chaos expansion: for any $c \in \mathbb{R}$,

$$1_{c \le Z} = \sum_{k=0}^{\infty} \alpha_k(c)\mathrm{He}_k(Z), \quad \alpha_0(c) = \Phi(-c), \quad \alpha_k(c) = \frac{e^{-\frac{c^2}{2}}\mathrm{He}_{k-1}(c)}{\sqrt{2\pi}k!}.$$

  The equality holds for all $Z \neq c$ and the convergence is uniform on compact set excluding c (Uspensky theorem).

Chaos expansion of the loss

- Denote the loss by $\mathcal{L} = \sum_{i=1}^{n} \ell_i \mathbb{1}_{\{a_i \epsilon_i + b_i \leq Z\}}$.
- Applying the chaos expansion :

$$\mathcal{L} = \sum_{i=1}^{n} \ell_i \sum_{k=0}^{\infty} \alpha_k (a_i \epsilon_i + b_i) \mathrm{He}_k(Z) = \sum_{k=0}^{\infty} \epsilon_{n,k} \mathrm{He}_k(Z),$$

where : $\epsilon_{n,k} = \sum_{i=1}^{n} \ell_i \alpha_k \left(a_i \varepsilon_i + b_i \right).$

- The truncation up to $I \in \mathbb{N}$ gives the I-chaos decomposition :

$$\mathcal{L}_I = \sum_{k=0}^{I} \epsilon_{n,k} \mathrm{He}_k(Z).$$

- There exists $C > 0$ s.t. $\mathbb{E}\left[|\mathcal{L} - \mathcal{L}_I|^2 \right] \leq C \frac{\left(\sum_{i=1}^{n} \ell_i\right)^2}{\sqrt{I}}$.
- The approximation is better for larger number of obligors n.

Conclusion

So far :

- Quantitative model of the impact of emission transition risk on credit risk, while accounting for the adaptation of the firm to climate policies
- Flexible model setup that takes future emission projection pathways as input and computes the associated default probability
- Large portfolio loss via Polynomial Chaos Expansion allows to significantly reduce computational time (many Gaussian characteristics can be computed off-line)

What's next?

- Needs for multidimensional PCE for different sectors (experiments in progress)
- Needs for physical risks too
- Needs for more accurate financial statements and balance sheet of the firm.

Thank you for your attention!