MATS352 Stochastic analysis
Excercise session 1

Monday, 24.3.2014

MaD 380 at 10.15

0.1 Write an essay of 2-4 pages on one (or both) of the following topics:

(a) Describe a problem that you hope to understand better after this course

(b) Explain why you want to learn stochastic analysis

You can hand in your essay at the exercise session or by email before the session (.pdf).

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1. Let $\Omega = [0,2)$ and define $\mathcal{F} = \sigma(\{t\} : t \in (1,2))$. Show that $[0,1] \notin \mathcal{F}$.

2. (a) Find two processes that are modifications of one another but not indistinguishable.

   (b) Find two processes (on the same probability space) that have the same finite-dimensional distributions but that are not modifications of one another.

3. Let X and Y be two processes with right-continuous paths, and assume that X is a modification of Y. Show that X and Y are indistinguishable. Is it enough to assume that almost all paths are right-continuous? What about left-continuous paths?

4. Let all paths of X be RCLL. For a fixed $t_0 > 0$, define

$$A_{t_0} := \{\omega \in \Omega\colon \text{ the function } t \mapsto X_t(\omega) \text{ is continuous on } [0,t_0)\}.$$

   Show that $A_{t_0} \in \mathcal{F}^X_{t_0}$.

5. Let all paths of X be LCRL. For a fixed $t_0 > 0$, define

$$A_{t_0} := \{\omega \in \Omega\colon \text{ the function } t \mapsto X_t(\omega) \text{ is continuous on } [0,t_0]\}.$$

   Assuming that X is adapted to a right-continuous filtration $(\mathcal{F}_t)_{t\geq 0}$, show that $A_{t_0} \in \mathcal{F}_{t_0}$.

6. Show that the compensated Poisson process (defined in Example 1.12) is a martingale w.r.t. its natural filtration. Which properties of the process did you need?