Scattering theory
Exercises #9, 03.12.2007
(return by 10.12.2007)

This is the last set of exercises.

1. Define the principal value of $1/x$ by

$$\langle \text{p.v.}\,\frac{1}{x}, \varphi\rangle = \lim_{\varepsilon\to 0}\int_{|x|\geq\varepsilon} \frac{\varphi(x)}{x}\,dx, \quad \varphi \in C_0^\infty(\mathbf{R}).$$

   Show that $\text{p.v.}\,1/x \in \mathscr{D}'(\mathbf{R})$ and that in the sense of distributions

$$\frac{d}{dx}\log|x| = \text{p.v.}\,\frac{1}{x}.$$

2. Show that in $\mathscr{D}'(\mathbf{R})$, one has

$$\frac{1}{x+i0} + \frac{1}{x-i0} = 2\,\text{p.v.}\,\frac{1}{x}.$$

3. If $a > -1$ define x_+^a by $x_+^a = x^a$ if $x > 0$ and $x_+^a = 0$ if $x < 0$. Show that $x_+^a \in \mathscr{D}'(\mathbf{R})$, and prove that

$$\frac{d}{dx}x_+^a = a x_+^{a-1}, \quad a > 0.$$

4. Prove the fact required in the proof of Corollary 2.3.14: if $\tilde{\mu} \in C^1(\overline{\mathbf{R}_+}; \mathbf{R}_+)$ and $(1+t)|\tilde{\mu}'(t)| \leq N\tilde{\mu}(t)$, $t \geq 0$, then

$$\frac{(1+s)^N}{(1+t)^N} \leq \frac{\tilde{\mu}(s)}{\tilde{\mu}(t)} \leq \frac{(1+t)^N}{(1+s)^N}.$$