Riemannian geometry
Exercises 4, 24.11.2015

1. Show that the path $t \mapsto (\cos t, \sin t, 0)$ is a geodesic on the Riemannian manifold $\mathbb{S}^2$.

2. Determine the maximal geodesics of speed 1 of the Riemannian manifold $\mathbb{S}^2$.

3. The mapping $F\colon B(0,1) \to \{x \in \mathbb{R}^2 : x_1 > 0\}$,

$$F(x) = \frac{(2x_1, 1 - \|x\|^2)}{x_1^2 + (x_2+1)^2}$$

is a diffeomorphism. Show that it is an isomorphism from the Riemannian manifold $\big(B(0,1), \frac{4\,g_E}{(1-\|x\|^2)^2}\big)$ to the upper haplfplane model $\mathbb{H}^2$ of the hyperbolic plane. The Riemannian manifold $\big(B(0,1), \frac{4\,g_E}{(1-\|x\|^2)^2}\big)$ is the *Poincaré disc model of the hyperbolic plane.*

4. Determine the maximal geodesics of speed 1 of the Poincaré disc model of the hyperbolic plane.

5. Determine the exponential map of the Poincaré disc model of the hyperbolic plane at 0.

6. Compute the length of the circle of radius r and the expression of the hyperbolic metric in normal coordinates with polar cordinates.

7. Show that a Riemannian manifold is connected if and only if any two points can be connected by a piecewise geodesic path.

[1] Vihje: Use the stereographic projection.
[6] Vihje: Use the Poincaré disc model.