You can also pass the course by a course work instead of exercises. Idea is to prepare a small written exposition on a specific topic, eg. a rigorous proof for Pontryagin max principle. Please contact the lecturer if you are interested in this option.

EXERCISES,
SELECTED TOPICS IN CONTROL THEORY, 2016
DEADLINE 26.02.2016

1. [1]Solve optimal time in the rocket railroad car model with initial point $(x_1, x_2) := (q, v) = (0, 1)$ using 'the heuristic method' in LN Example 1.6.
2. Justify why the discount factor is of the form $e^{-\lambda s}$ in the infinite time horizon problem.
3. Give a more realistic investment planning (cf. LN Example 1.3) by incorporating a terminal payoff at time T.
4. Suppose you model the value of the company (cf. Exercise 3) by the terminal payoff $g(x(T)) = 2x(T)$. Solve for the optimal control. Why is the solution not surprising?
5. (2 points) Solve the optimal control for the production planning example in Case 1 ie with state constraint $x(s) \geq 0$ and no final cost (see LN Example 1.3).
6. Try the dynamic programming/feedback approach (LN Sec 2.1.1) for the three velocities problem (cf. LN Example 2.9) starting inside Region I assuming you are given the value u.
7. Same as Exercise 6 but starting inside Region II. What possible problems do you observe?
8. Show that the value obtained in Example 2.8 in LN is a viscosity solution to $u_t - |u_x| = 0$.
9. Show that if u is a viscosity solution to $u_t + \inf\{r + Du \cdot f\} = u_t + H(x, Du) = 0$ then $v = -u$ is a viscosity solution to $v_t + \sup\{-r + Dv \cdot f\} = 0$.
10. (2 points) Show that the control given by the Pontryagin max principle to the reinvestement planning/control of production consumption (see LN Example 2.13 or ELN Example 4.4.2) is essentially unique.
11. Consider, under the usual regularity assumptions, the problem of maximizing $\int_t^T r(x(s), \alpha(s))\, ds$, and assume that the problem is abnormal i.e. the Pontryagin max principle only holds for control Hamiltonian $H(x, p, a) = f(x, a) \cdot p + 0 \cdot r(x, a)$. Show that then any control satisfies the Pontryagin max principle equations.
12. Formulate the Pontryagin max principle equations for the rocket car problem.

[1]LN below refers to course lecture note, and ELN to Evans' lecture note.

13. Consider the problem of minimizing the exit time from $B(0,1)$ i.e. minimize
$$\int_0^\tau 1\,ds$$
where $\tau = \tau(x,\alpha)$ is the first exit time from $B(0,1)$, $x \in B(0,1)$, and dynamics is given by $\alpha(s) \in \overline{B}(0,1)$ and
$$\begin{cases} x'(s) = \alpha(s) \\ x(0) = x \end{cases}$$
Write down the optimal control, value function, and the corresponding Hamilton-Jacobi PDE.
14. Show that the value function in Exercise 13 is a viscosity solution to the corresponding Hamilton-Jacobi PDE.
15. It was shown in the lectures that the rocket car example has an optimal bang-bang control (cf. LN Example 6.5 or ELN Thm 3.1). In the three velocities example, Region II, we know that optimal control is not bang-bang. Point out where the proof that worked for the rocket car fails for the three velocities example (even if you take $\alpha : [t,1] \to [-1,1]$ controls in the three velocities example).
16. Justify by a formal calculation the HJB in ELN Application 7.6, (7.19).
17. (2 points) You are Secretary of the Treasury and use the following model for state economy
$$\begin{cases} x_1'(s) = (\alpha(s) - d)x_1(s) \\ x_2'(s) = (1 - \alpha(s))x_1(s) + rx_2(s), \end{cases}$$
where $x_1(s), s \in [0,T]$ is tax income rate of the state and $x_2(s)$ is national debt, and $d, r \in (0,1)$ are given constants $r << 1$, $x_1(0) > 0 > x_2(0)$. You want to maximize $x_1(T) + rx_2(T)$. Solve for the optimal control α. Suggest improvements.