JYVÄSKYLÄN YLIOPISTO

MATEMATIIKAN JA TILASTOTIETEEN LAITOS

Exercise set7 **Topological vector spaces**

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7.1. *Let E locally convex Hausdorff-space and $A \subset E$. Find a necessary and sufficient condition for that the polar of A:n in E^* (or E') is $\{0\}$. (Some help:: bipolaari?)*

7.2. *Let E be a locally convex Hausdorff-space and $B \subset E$ balansoitu, konveksi and bounded – and complete. Define Therefore is a seminormi, since B is balansoitu, konveksi and in E_B s absorboiva. In fact the gauge p is a norm, since B is bounded in tehe original Hausdorff-topology τ of E , so we write $p = \|\cdot\|$. Is the unit ball of E_B same as the closure $\bar{B}$?*

7.3. *Let E be a locally convex Hausdorff-space. Then (E, E^*) is separoiva. Therefore the completion of $E_{\sigma(E,E^*)}$:is the algebrallinen duaali $(E^*)'$. Can E_σ be complete?*

7.4. *Let (E, F) separoituva dualiteetti and $M \subset E$ vector subpace. Prove that $M^{\perp\perp} = M$ if and only if M is closed in some compatible topology wrt duality (E, F).*

7.5. *Prove that*

a) $\mathfrak{S}-$topology is locally convex and is given by the gaugesof the polars of the $A \in \mathfrak{S}$, $p_A(y) = \sup_{x\in A} |\langle x, y\rangle|$.

b) if $\mathfrak{S}$ satisfies the conditions

(1) $A, B \in \mathfrak{S} \implies \exists C \in \mathfrak{S}$ *such that* $A \cup B \subset C$ *and*

(2) $A \in \mathfrak{S}$, $\lambda \in \mathbb{K} \implies \exists B \in \mathfrak{S}$ *such that* $\lambda A \subset B$,

then $\{A^\circ \mid A \in \mathfrak{S}\}$ is a basis of neighbourhoods of the origin in the $\mathfrak{S}-$topologiy.

c) If $\mathfrak{S}$ satisfies $\bigcup_{\mathfrak{S}} A = E$, then $\mathfrak{S}$-topologia is finer than weak topology $\sigma(F, E)$.

7.6. *Prove directly (No Alaoglun and Bourbakin), that anequicontinuous set $A \subset E^*$ is weakly bounded .*

7.7. *Let E non-complete locally convex Hausdorff-space and $\hat{E}$ its completion. Prove that the topology $\sigma(E', \hat{E})$ is strictly finer than $\sigma(E', E)$ andsimilarly for E^*.*

Solution: The weak topologia is compatible , so the dual of E' : is E in one topology and $\hat{E}$ in the other. The topologies must be different!

7.8. *Let E be a Banach space. Prove that $b(E, E') = \tau(E, E')$.*

7.9. **Is Schwartzin testifunktiospace $D(\mathbb{R})$ normable? How about the spaces $D(K)$?*