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Undergraduate Representation Theory 2010 **Exercise Set 1**
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Reading: Dummit and Foote *Abstract Algebra* (2004) pp 1–6, 8–11, and 16–21.

Problem 1. A study of D_4.

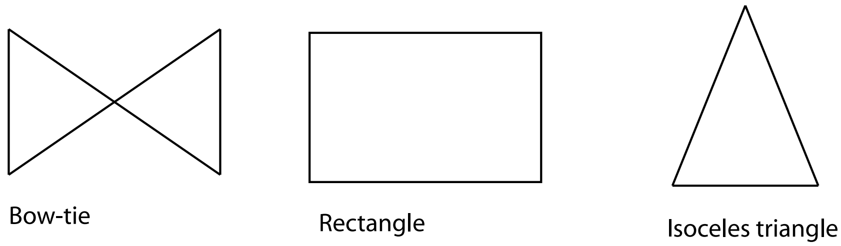
- Complete the multiplication table for the group D_4 of symmetries of the squa-re.

$\circ$	e	r_1	r_2	r_3	A	H	D	V
e								
r_1								
r_2								
r_3								
A								
H								
D								
V								

- Find a natural set of *generators* for D_4. That is, find a set of symmetries of the square such that every symmetry in D_4 can be obtained by iterating these given ones (as many times as we like, and in any order). What is the smallest possible number of generators D_4 can have?
- Note that some symmetries of the square preserve orientation and some do not. How many of each type are there? Does the set of all orientation preserving symmetries form a subgroup of D_4? Does the set of all orientation reversing symmetries form a subgroup of D_4?
- Suppose that sides of the square are colored either red or blue, so that opposite sides have the same color. Describe the symmetry group of the colored square as a subgroup of D_4.
- Describe all subgroups of D_4. How many are there?

Problem 2 Describe the symmetry groups of the following figures:

Problem 3 Without tediously writing out the multiplication tables, how much of Problem 1 can you generalize to the group D_n of symmetries of the regular n-gon? For example, what is the order of D_n? Is it abelian? Can you find a natural set of generators? What is the smallest possible set of generators? What natural subgroups can you identify?



Problem 4 In any group G, show that the identity element is unique. Show also that each element $g \in G$ has a unique inverse g^{-1}.