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SYKE

# ESTIMATING YEARLY NUTRIENT FLUXES VIA GAUSSIAN STATE SPACE MODELS

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## ABSTRACT

**Reliable estimates of the nutrient fluxes carried by rivers from land-based sources to the sea are needed for efficient abatement of marine eutrophication. The infrequent data calls for ways to reliably estimate the nutrient concentrations of the missing days. Here we use the Gaussian state space models with daily water flow as a predictor variable to predict missing nutrient concentrations and yearly fluxes for four agriculturally impacted Finnish rivers. Our results show no trends in yearly nutrient fluxes over the last 25 years for all of the four rivers examined.**

## MOTIVATION

Abatement of marine eutrophication calls for reliable estimates of the nutrient fluxes carried by rivers from land-based sources to the sea. Monitoring programs of rivers typically involve daily measurements of water flow, but due to the costs, much more infrequent sampling of phosphorus and nitrogen concentrations.

Our primary interest is a yearly nutrient flux, denoted by

$$m_{t,s} = \sum_{i=1}^{s} q_{t+i} c_{t+i}$$

where $q_t$ is the water flow on the day $t$ and $c_t$ is the daily nutrient concentration. Neglecting the measurement errors, if we had the values $q_t$ and $c_t$ measured on each day, then we would have correct nutrient fluxes.

## DATA

Our data consist of the concentrations of total phosphorus and total nitrogen, and daily water flow measurements from four rivers located in southern Finland, Aurajoki, Paimionjoki, Porvoonjoki and Vantaanjoki, during 1985–2010. Daily measurements on nutrient concentrations are available for only 5–10% of the time period.

## METHODS

For the prediction of the missing phosphorus and nitrogen concentration measurements we use a Gaussian state space model which consists of first order autoregressive component, and time varying regression components with daily water flow measurements as a predictor variables. The graphical representation of this model is shown in Figure 1. As the phosphorus and nitrogen concentration measurements are correlated, we model them together but separately for each river, allowing the observational disturbances $\varepsilon_t$ and the state disturbances $\xi_t$ to correlate between nutrient-specific models.

The unknown parameters of the model are estimated by maximum likelihood method. As a special case, when autoregressive coefficient $\phi$=0 and var($\eta_t$)=0, an ordinary regression model is obtained. Given the estimated model, we estimate the yearly fluxes via conditional simulation of missing nutrient concentrations.

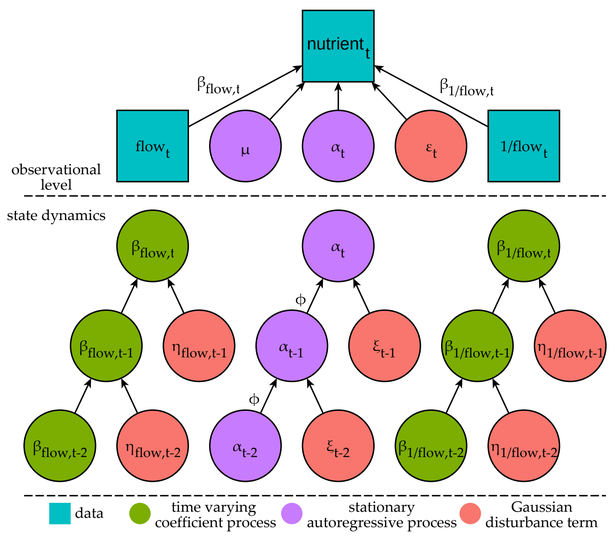


*Figure 1. Gaussian state space model for the single nutrient concentration series.*

## RESULTS

The yearly estimates of nutrient fluxes with their simulated 95% prediction intervals are shown in the Figure 2. Each river exhibits similar fluctuating patterns without a clear trend. Especially yearly phosphorus fluxes, but also nitrogen fluxes clearly peak in 2008, followed by an even larger drop in 2009. The effect of varying sampling frequency is visible in the widths of prediction intervals.

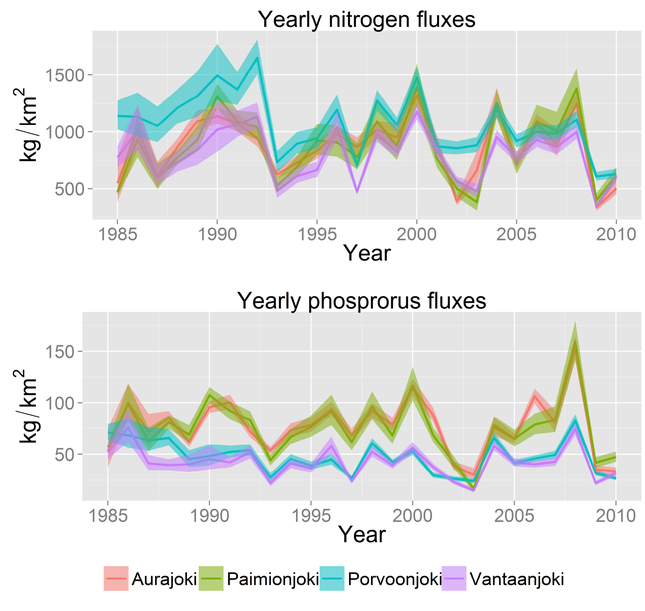


*Figure 2. The point estimates and 95% prediction intervals for the yearly nutrient fluxes.*

## COMPARISON TO ORDINARY REGRESSION

Based on the simulation experiments [1], ordinary regression model gives somewhat biased flux estimates and produces the coefficients of variation that are often substantially smaller than our time varying model. As can be seen from Figure 3, the coefficients of variation from our model depend on the yearly sample sizes, whereas results from the ordinary regression model are overoptimistic and counterintuitive: uncertainty in the yearly flux estimate is independent from the amount of measurements in a given year.

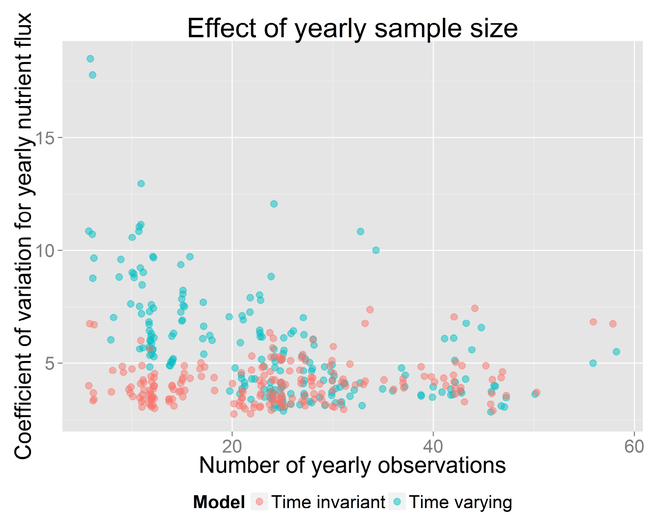


*Figure 3. The relationship between coefficients of variation for yearly nutrient flux and the number of yearly nutrient concentration observations.*

## CONCLUSIONS

**During the observational period 1985–2010, Finnish agricultural farmlands experienced a substantial decrease in phosphorus and nitrogen balance [2]. Despite this drastic decrease in nutrient balance, we did not observe any corresponding trends in nutrient fluxes over the last 25 years for any of the four rivers examined.**

**Our results seem to indicate that when daily flow data are available, relatively sparse data on nutrient concentrations can be used to estimate yearly fluxes, when the dynamic aspect of the phenomenon is taken into account.**

## ACKNOWLEDGEMENTS

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## REFERENCES

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